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# «ЗОЛОТИЙ ПЕРЕТИН» В ПРЕЛЮДІЯХ ДЛЯ ФОРТЕПІАНО Ор. 8, № 1–4 В. БАРВІНСЬКОГО<sup>1</sup>

В цій роботі зроблено унікальний гармонійний аналіз кількох порівняно коротких музичних композицій. Як приклад взято фортепіанну прелюдію всесвітньо відомого українського композитора XX століття Василя Барвінського. Метою цього аналізу є розкриття математичних закономірностей у структурі завершеної музичної композиції, зокрема відповідності її композиційної структури пропорціям «Золотого перетину». Таким чином, підтвердження зазначеної вище відповідності (тобто математичної закономірності) підтримує теорію універсальності математичної гармонії в мистецтві.

Ключові слова: золоті пропорції, реалізація гармонії математики в музиці.

Золотий перетин – це співвідношення, що описує точні фізичні пропорції багатьох біологічних структур, що зустрічаються в природі, а також пропорції, які люди схильні вважати найбільш естетичними в багатьох мистецьких творіннях (Стахов, 2006). Золотий перетин також виявляється в межах як просторових, так і часових сферах музичних творінь, будь-то в усних народних піснях або в нотних творах високого музичного мистецтва. У статті досліджується поява Золотого перетину у фортепіанних прелюдіях № 1-4, Ор. 8 українського композитора Василя Барвінського (1888—1963). Точки золотого перетину обчислюються в межах музичних структур його прелюдій із теоретичним визначенням їх значущості. У цьому документі робиться висновок, що фортепіанні прелюдії Барвінського, № 1—4, містять музично значущі моменти для слухача саме у випадках Золотого перетину.

Оскільки джерела інформації про Барвінського, серед яких і його мемуари, не розкривають причин чи можливі пояснення, чому в роботі Барвінського з'являються пропорції «Золотого перетину», то деякими можливими поясненнями можуть бути: (1) збіг, (2) підсвідома або естетична схильність, (3) свідоме наслідування Західної музики, (4) свідоме наслідування народної музики, або їх поєднання. Деякі композитори — Аренський, Бах, Барток, Бетховен, Шопен, Дебюссі, Гайдн, Моцарт і Шуберт — мають твори, в яких присутні пропорції "Золотого перетину". В роботах ряду дослідників є деякі пояснення щодо певних композиторів, однак щодо Барвінського це питання залишається відкритим. Проте зібрані докази свідчать, що подані вище можливі пояснення наведені в порядку їх вірогідності, тобто, найменш можливим є збіг, а найімовірніше — наслідування народної музики.

**Висновки**. Ми впевнились, що всі чотири прелюдії містять музично видатний матеріал, що знаходиться в точках Золотого перетину або безпосередньо перед ними чи після них. У дуже загальних рисах можна сказати, що точки золотого перетину збігаються з кульмінацією першої прелюдії, а також з її найбільш спокійною частиною; у другій прелюдії — з місцями найбільшого діапазону, текстурного і ритмічного контрасту; у третій прелюдії — з найбільш драматичними і контрастними частинами;

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<sup>&</sup>lt;sup>1</sup> Передрук з Nina Soyfer. The Golden Section in Barvinsky's Piano Preludes Op. 8, Nos. 1–4. *VisMath*. 2015. Publisher: Mathematical Institute SASA, Belgrade. ISSN: 1821–1437. Стаття подається в оригіналі з незначними скороченнями. Надіслано автором.

і у четвертій прелюдії – з її кульмінацією, а також її текстурно контрастною точкою. Крім того, в прелюдах Барвінського ключові зміни, зміщення півтонів у тональності та взаємодія з ритмічним потоком часто оточують точки Золотого перетину.

Багато візуальних митців — художників, архітекторів і скульпторів, як відомо, свідомо використовували пропорції Золотого перетину у своїх роботах. Незважаючи на поширеність цього прийому в творах Барвінського, я не знайшла жодного документального підтвердження, яке б свідчило про свідоме його використання. В жодній літературі, доступній автору, також не згадується будь-який зв'язок між творами Барвінського і цією математичною пропорцією. Бахман писав про Белу Бартока: "По суті, здається, що використання Бартоком чисел Золотого перетину і Фібоначчі просто прояснює і фокусує винятковий баланс і пропорцію, які існують у його музиці" (Васһтапп 1979, 80). Подібне можна сказати і про Барвінського, чи слід шукати більш чіткі пояснення?

Справжня мета представленого дослідження полягає не в тому, щоб довести знання Барвінським математики або законів природи і фізики, але, щоб проілюструвати, що він може бути в когорті видатних композиторів і талановитих музикантів, оскільки Золотий перетин також присутній у творах таких великих композиторів, як Бах і Шопен (але бракує у творах деяких інших композиторів, а це вивчав Сабанєєв).

Я б хотіла завершити двома постулатами. Перший постулат полягає в тому, що, можливо, Золотий перетин, що використовується несвідомо, може служити критерієм якості твору мистецтва. Хоча в прелюдіях Барвінського практично не існувало прямого зв'язку між точками наслідування фольклору та Золотого перетину. Моя друга думка полягає в тому, що досконале знання народної музики (а цим Барвінський є знаменитим) може бути пов'язане з використанням або сприйняттям композитором Золотого перетину, що формує відкрите питання і можливість подальших досліджень.

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# 1. Introduction:

The Golden Section is a ratio describing the precise physical proportions of many naturally occurring biological structures as well as the proportions which humans tend to find most aesthetically appealing within many artistic creations (Stakhov 2006). The Golden Section has also appeared within both the spatial and temporal domains of musical creations, whether in aurally transmitted folk songs or in the notated works of Western art music. This paper examines the appearance of the Golden Section in the *Piano Preludes Nos. 1-4, Op. 8* of Ukrainian composer Vasyl Barvinsky (1888–1963). Golden Section points are calculated within the musical structures of his preludes with a speculative offering as to their significance. This paper concludes that Barvinsky's *Piano Preludes, Nos. 1–4*, contain musically significant moments for the listener precisely at, or immediately following, incidences of the Golden Section.

As sources of information about Barvinsky, including his memoirs, fail to reveal the cause, some possible explanations as to why Golden Section proportions appear in Barvinsky's work are (1) coincidence, (2) subconscious or aesthetic predilection, (3)

deliberate imitation of Western art music, (4) deliberate imitation of folk music, or a combination thereof. Some Western art music composers exhibiting Golden Section proportions in their work and hence subject to the same question are Arensky, Bach, Bartok, Beethoven, Chopin, Debussy, Haydn, Mozart, and Schubert. While the question has been answered by researchers in some cases, the question remains open in Barvinsky's case. However, evidence collected thus far suggests that the possible explanations above are stated in order of their likelihood, i.e., coincidence being least likely, and folk music imitation being most likely.

# 2. Golden Section and Barvinsky

When analyzing Barvinsky's *Preludes* in detail, it becomes apparent that they are well crafted and harmonious. Considered from the point of view of contemporary theories of mathematical harmony, it becomes clear that their structure obeys the law of harmony, as determined by the Golden Section. Claudio Ptolemy (or Claudius Ptolemaeus) first introduced the term, though it was popularized by Leonardo da Vinci, after which the term came to be known as the *da Vinci Code*. [...]

The Golden Section can be geometrically understood using a single horizontal line. Divide a given line-segment AB at a point C where the longer segment AC relates to the shorter segment CB as the whole line AB relates to the longer segment AC. If this proportion were written in the form of an equation, where  $\tau$  equals the sum of the lengths of the two segments divided by the length of the longer segment, the resulting  $\tau$  would be a number which is one more than its reciprocal, or the irrational number 1.6180339887498948482045868343656... The Golden Section of a quantity can be obtained by dividing that quantity by this number  $\tau$ , approximately  $1.618.^2$ 

$$\frac{AB}{AC} = \frac{AC}{CB} \qquad \frac{C}{A}$$

# 3. The Golden Section in Prelude 1, Op. 8

Let us view the horizontal geometrical line as the time line, or the length of, for instance, Barvinsky's *Prelude* 1, op. 8, calculated in bars. This prelude is 75 bars long. Let this be the length **L** of segment *AB* above. The unknown Golden Section of this prelude occurs at a point *C* which can be determined using the approximate value of  $\tau$ , 1.618. This allows for calculation of the position **P1** of the Golden Section ( $\mathbf{L}/\tau$ ) in this *Prelude*: at a point in bar 46 (75 ÷  $\tau \approx 46.353$ ).

Coming back to the music, bar 46 marks precisely the beginning of the second section of its fourth variation that happens to be this prelude's dramatic climax (end of bar 46 to bar 58). It contains the <u>densest texture</u> (up to six notes in the melodic line, with up to five-octave range, and up to five note chords constantly filling the space between melodic lines, with up to five chords per melodic note), agitated and expressive musical <u>character</u> (e.g. composer's marks: *passionato ben marcato* in bar 52, *sempre ff* in bar 46-47), *accents* on octaves of high register (bars 49, 52, 53), and the use of

<sup>&</sup>lt;sup>2</sup> The Golden Section is also commonly referred to by the Greek letter  $\varphi$  (phi).

unresolved and strongly altered <u>harmony</u>. The material is technically challenging and written in four parts per two hands. In his mathematical papers on the Golden Section and music, Stakhov notes that in many Western music masterpieces, the Golden Section coincides with the climax.<sup>3</sup> Another outstanding feature of this point in Barvinsky's *Prelude* 1 is a <u>semitone shift of tonality</u>.<sup>4</sup> The key of G sharp major is suggested in bars 46-47 and is fully established in bar 48 (see **Figure 2**). The transition is very sudden, although the tonic of G major in its second inversion could serve as enharmonic sharp seven's triad of G sharp major. In any case, the G sharp major in bar 47, where the F\*, E\*, D\*, and C\* first emerge, apart from transferring the music to the new key, initiates the episode of this prelude's climax.

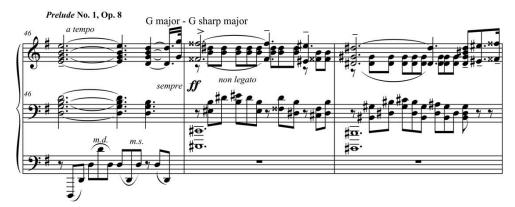


Figure 2: Prelude 1, bars 46-48, semitone tonality shift

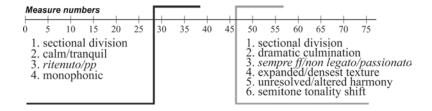
Moreover, if we look at the opposite division of the prelude, where B is the beginning and A is the end, then a symmetric Golden Section point P2 ( $L - L/\tau$ ) occurs at bar 28.647 (75 – 46.353), or the second half of bar 28. This exact place, interestingly, is the calmest and most tranquil place of the entire prelude. The second half of bar 28 contains a single line of six eighth notes. Firstly, the <u>texture</u> is the <u>thinnest</u>. Secondly, all the sounds are of <u>consonant</u> nature (arpeggiated sounds of B seventh chord, or mediant seventh of G major). Lastly, tranquility is expressed through slow motion, since the ritenuto marking is indicated in bar 28.

In the remainder of this paper, the length of each prelude analyzed is labeled L, and the Golden Section points corresponding to  $L/\tau$  and  $L - L/\tau$  are labeled P1 and P2, respectively. Note, however, that P2 always occurs before P1. As visual aids to the analyses of this and the following preludes, the locations of Golden Section points P1 and P2 along the time line are provided in Figures 3, 6, 10, and 12. The horizontal time line is divided uniformly into bars. However, since not all of the preludes' 2 and 3 bars have the same duration, P1 and P2 are calculated in eighths for prelude 2, and in

<sup>&</sup>lt;sup>3</sup> For instance, Stakhov draws on Sabaneev's discussion of Chopin's études and use of the Golden Section in them (Stakhov 2009, 113).

<sup>&</sup>lt;sup>4</sup> This compositional technique is an outstanding and characteristic feature in Barvinsky's music (Soyfer 2012).

sixteenths for prelude 3. Below is a summary of outstanding musical features mentioned in the preceding analysis of prelude 1, along with positioning of **P1** and **P2** on a temporal audiogram, created using *Cool Edit Professional* (see **Figure 3**).



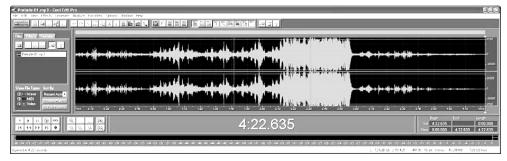


Figure 3: Prelude 1, Golden Section points P1 and P2, diagram and audiogram

In the figure above, the two Golden Section points on the score-based time line and the corresponding points on the audiogram do not exactly coincide vertically. This is due to non- uniform tempo within the audiogram of the prelude's live performance by Lviv pianist and Professor Maria Krushelnyzka (Krushelnyzka 2006). Specifically, her introduction and especially ending are in slower tempo than the middle, thereby moving both **P1** and **P2** toward the center of the time line as shown.

### 4. The Golden Section in Western Art and Folk Music

Works of prominent composers such as Arensky, Bach, Bartok, Beethoven, Chopin, Debussy, Haydn, Mozart, and Schubert are seen, upon analysis, to employ the Golden Section as Barvinsky does. "In the 1,770 musical works of 42 composes that were studied by him [Sabaneev], the golden mean <sup>5</sup> is met 3,275 times. The greatest frequency of musical works based upon the golden mean is found in Beethoven (91%), Haydn (97%), Arensky (95%), Chopin (92%), Mozart (91%), Schubert (91%), and Scriabin (90%)" (Stakhov 2009, 114). In their paper on Golden Section and Fibonacci Numbers, <sup>6</sup> Tibor Bachmann and Peter J. Bachmann stated that "...there are numerous examples in Bela Bartok's music where the Golden Sections and Fibonacci numerical sequences are

<sup>&</sup>lt;sup>5</sup> The Golden Section is also commonly referred to as the Golden Mean.

<sup>&</sup>lt;sup>6</sup> The Fibonacci Sequence of Numbers is 0,1,1,2,3,5,8,13,..., where the next number in the sequence is defined as the sum of the previous two. The ratio of consecutive Fibonacci Numbers approaches the Golden Section in the limit.

used as a structural foundation" (Bachmann 1979, 72). There is a debate over whether this was done deliberately as a compositional device in all cases, or whether it was coincidental in some instances. For example, Chopin's *Fantasy Impromptu* contains the Golden Section, but being impromptu and not subject to editing, the Golden Section division could not have been applied deliberately by Chopin (Stakhov 2009, 115). Also, in his study of Mozart's piano sonatas, John F. Putz concludes that, "Mozart may have known of the Golden Section and used it. That there is considerable deviation from it ... suggests otherwise, however" (Putz 1995, 281). A similar conclusion was made by Bachmann about Bartok's music (Bachmann 1979, 81). Likewise, the occurrence of Golden Section divisions in folk music is not likely to have been placed by careful calculation. As for Barvinsky, I have found neither mention nor evidence of mathematical knowledge or Golden Section calculation on his part in any literature.<sup>7</sup>

In our conversation, Dr. Alexey Petrovich Stakhov noted an inexplicably high quantity of Golden Section appearances at significant moments in folk songs. He considers folk song refined through oral tradition as an art form. This art form was one of Barvinsky's major influences. As documented by himself and other specialists of his life and work, Barvinsky listened to, studied, and incorporated the folklore of not only Ukraine, but also of other countries, into his work over the course of his entire compositional career. According to Barvinsky's memoirs, he started his compositional practice by arranging folk songs in various choral forms. My previous research showed that his *Piano Preludes* also contain several elements of Ukrainian folklore (Soyfer 2010). There is, thus, a possibility (although without statistical/empirical prove) that direct experiential knowledge of folklore and/or older forms of orally transmitted music instilled a tendency to compose pieces with musical sections of similar proportions to those found in folk songs.

Coming back to the Golden Section within Barvinsky's piano preludes, I would like to look at preludes 2, 3, and 4 through similar lenses. Particularly, where do Golden Section points occur within their timelines, as expressed sequentially through the notes and bars, and do these points occur at or near musically significant or outstanding material?

The following will be divided into three sections, one for each prelude. Within each prelude's discussion, the *calculations* subsection will locate the Golden Section points mathematically, and the *analysis and significance interpretation* subsection will expand on these points' significance within preludes' basic structures (e.g. within form, tonality, harmony).

# 5. Prelude No. 2, Op. 8

### Calculations:

This prelude contains **61** bars. Since its 6/8 time signature varies to 3/8 for one bar, let us view the horizontal time line of this prelude in eighth notes, or in beats within bars, rather than in bars. Thus,  $\mathbf{L} = 360$  (60 bars of 6 beats) + 3 (1 bar of 3 beats) = **363** beats in total. The Golden Section points are **P1**: beat **224** (363  $\div \tau \approx 224.351$ ), and **P2**:

<sup>&</sup>lt;sup>7</sup> The literature studied exceeds the list of related writings in this paper's bibliography

the second half of beat 138 (363 – 224  $\approx$  138.649). **P1** occurs at the penultimate (fifth) eighth of bar 38, and **P2** occurs at the final (sixth) eighth of bar 23.

# Analysis and Significance Interpretation:

The *form* of this prelude is based on theme and variations, or variational development. The prelude can be divided as follows: introduction (bar 1), theme (a binary sentence in bars 2-9), four variations (1st – bars 10-15, 2nd – bars 16-22, 3rd – bars 31-38, 4th – bars 51-57), two episodes (or transitional sections, 1st – bars 23-30, 2nd – bars 39-51), and a coda (or lento section – bars 58-61). The prelude is written in F sharp major. The range of *tonality* is quite narrow, with modulatory keys such as iii (A sharp minor), bII (G major), V (C sharp major), and ii (G sharp minor). The *harmony* is standard and unusual at the same time. The customary V-I and II-IV-V-I cadential harmonic progressions are often employed (e.g. bars 36-37, 57-60), however, the chromatic nonfunctional or passing chords add special harmonic colouring and impressionistic feel (when they are also parallel) to the prelude. Three harmonic devices in this prelude contributing to Ukrainian folk sound are: 1) prevailingly plagal harmonies and modulations, 2) use of parallel sixth chords as in bars 6-7, and 3) pedal points (Arkhimovych et al. 1964, 54-55). Among these features, plagal modulation follows **P1**, and pedal point surrounds **P2**.

What is musically outstanding about the Golden Section points in prelude 2? **P1** appears at the end of bar 38, on the border of the third variation (theme-based material) and the second episode (new material), thus marking a <u>sectional division</u>. [...]

This point occurs immediately before bars 39-40, which are rhythmically outstanding in that they employ groups of three eighth notes in the left hand and four subsequent dotted quarters in the right hand, features seen nowhere else in the prelude. These bars lead into the choral-like section in bars 41-48. It is the only section in this prelude that is syncopated, homorhythmic, in low register of the piano, with expansion of time values, and it has the prelude's lowest note density (two co-soundings per bar). The note density (horizontally) reduces from 4-14 notes per bar in bars 1-40 to 1-3 notes per in bars 41-48. The two bars 41-42 are the least active in the entire prelude, with poco a poco rilasciando (from bar 39), pianissimo dynamics (bar 41), and sustained supertonic chord. The rhythmic structure of the choral section resembles the reverse of the rest of the prelude's typical <u>left hand</u>, where longer and shorter sounds alternate as quarters and eighths. Instead, in bars 41-48 shorter sounds are alternated with longer. For instance, in bars 45-46, a quarter is followed by a half note. Being homorhythmic in texture, this section abandons the sixteenth-note melody altogether (seen in most of the prelude), leaving only vertical sustained and syncopated chords. P1 precedes chords that are tied through bar lines and suspended, breaking the rhythmic flow of the prelude. In fact, every bar before bar 41 had clear divisions in groups of three eighths in that every bar had a note on first and fourth eighth, whereas the choral section blurs this pulse.

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<sup>&</sup>lt;sup>8</sup> Although it could be argued that prelude 2 contains elements of through composed form and binary form, this analysis considers it to be in theme and variations form, in accordance with Barvinsky's annotated list of compositions (Barvinsky 2004).

As for tonal and harmonic structure, the second episode or material immediately following **P1** is interesting in its modulation to supertonic key. F sharp major harmony in bars 38-39 is followed by modulation to G sharp minor. As most of this prelude's modulations, this modulation in bar 40 is plagal, in that one or both of its pivot chords are of subdominant function. An interrupted cadence (dominant in its second inversion followed by submediant triad in the beginning of bar 40) precedes F sharp major's dominant in the second half of bar 40, which serves as G sharp minor's subdominant. A number of suspensions and sustained chords occupy bars 41-46. These include supertonic, submediant seventh, mediant, and finally dominant seventh chord in its second inversion. The resolution of this chord in G sharp minor's tonic also serves as the first chord of F sharp major (minor supertonic triad), followed by its dominant ninth and half diminished seventh, written out as an ascending passage of sixteenth thirds, where each non-functional third acts as an incomplete auxiliary third (bars 49-50). This passage concludes the choral section and leads into the next variation.

The second Golden Section point **P2** in bar 23 appears almost exactly at another sectional division between the second variation and the first episode (see bars 22-23). Moreover, **P2** occurs exactly in the first bar of the chromatic ascending sequence (bars 23-26) leading into the prelude's most melodically expanded passage in bars 27-30 (see **Figure 5**).



**Figure 5:** Prelude 2, bars 23-30, grey section highlighting Golden Section point **P2** in bar 23

These bars (23-30) are also unique to the prelude in that they contain a section of the greatest tempo flexibility within the prelude: from a tempo, to poco allargando, to accelerando, and to rallantando – within only 8 bars. The sequence is rhythmically outstanding in that it is the only place where rhythmic figure formed by a sixteenth rest (instead of dotted quarter sustained from previous bar) and five sixteenths is employed exactly five times in the right hand (bars 23-26). Another unique element is the symmetrical arc (bell) shape of this rhythmic figure's melody, with middle note being the highest (see bars 24-26). In terms of sequential structure, as it will be seen in the third prelude as well, Barvinsky employed a stretto technique in that the last two of five sequential units are half the size of the first three, only a half bar each (see bar 26). In fact, this device brings the melodic shape in the right hand closer to the passage's melodic shape. The first seven notational symbols of bar 27, which form ascending units of the passage, noticeably resemble the sequential five-note melody. This melodic development assists in achieving the gradual transition from sequence to passage. The passage (bars 27-30) is outstanding rhythmically in that nowhere else in the prelude does the right hand contain four bars of constant sixteenth notes. In most of the prelude, melody in the right hand is alternated with prolonged stops (e.g. bar 16). In bars 27-28 the highest rhythmic contrast between the two hands takes place: left hand sustained for three bars on a single three-note chord: C# of the big octave, G# of the big octave, and E# of the small octave; against the aforementioned rapid sixteenth motion in the right hand. This passage ends with bar 30, which is twice as short as all the other bars in the prelude (3/8 instead of 6/8). This not only creates an effect of rhythmic shift, but also reverses the feel of strong and weak halves of each bar, or pulse reverse. Finally, the same eight bars following **P2** contain the largest <u>range span</u> within this prelude – over five octaves: from C# of the big octave to D# of the fourth octave (see bar 28). Thus, **P2** presents unique melodic, rhythmic, textural, and structural qualities.

Preceding **P2**, the second variation gradually brings the prelude to lower register, slower pace (*poco rit.* in bar 21), narrow range (one octave), and softer dynamics (*poco dim.* in bar 20). The dynamic indication in bar 23 is triple piano *ppp* and *a tempo legiero*. The melodic line starts with A of the small octave, the lowest note of the right hand's melodic line. Immediately after the Golden Section a reverse in music direction takes place with rapid ascending figures, *stretto* development, widening of range, and, finally, expanded passage. Bars 23-25 are united by consistent C\$\pi\$s in the bass which could be viewed as <u>pedal</u> of the first scale degree or a kind of drone. At the same time the first episode is <u>harmonically and tonally unstable</u>. For example, the dominant seventh of C sharp major with flat five, in bar 23, can be seen as a non-functional passing chord, connecting the preceding flat supertonic and the following tonic. The flat supertonic in bar 24 can be considered an auxiliary chord ascending in secondary dominant sixth of C sharp major. Similarly, although the first chord of bar 25 (consisting of E, G\$\pi\$, B, and C\$\pi\$) could be viewed as minor dominant seventh of F sharp major, it is perhaps an auxiliary non-functional chord which descends into another secondary dominant of C

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<sup>&</sup>lt;sup>9</sup> When referring to an auxiliary non-functional chord, the author means the chord in which all sounds serve as auxiliary notes of the following functional chord. It may also be called an ornamenting chord (such as common tone sevenths).

sharp major's dominant, a seventh in its third inversion. Bar 26 starts with another C-sharp minor seventh, which functions as F sharp major's minor dominant seventh (E $^{\ddagger}$ ) in its root position. In the second half of this bar E $^{\ddagger}$  becomes E $^{\ddagger}$ , forming the major dominant seventh chord of F sharp major. This harmony remains within the sustained chord in the left hand (C $^{\ddagger}$ -G $^{\ddagger}$ -E $^{\ddagger}$ ) and in the passage of sixteenth-notes in the right hand (spanning from B of small octave to D $^{\ddagger}$  of the fourth octave) until the end of bar 30, resolving in home key by the next bar.

Both Golden Section points appear on the border of a variation and an episode. It is fascinating to me that there are only two episodes in the prelude, and each starts at a Golden Section point! At first glance, the first and the second episodes have little in common. Upon closer examination, however, similarities in their function become apparent. For instance, each episode introduces new musical material contrasting with the rest of the prelude (e.g. rhythmically and texturally); moreover, each episode ends with a fast sixteenth-note passage that leads into the next variation. In describing the significance of these two points in terms of temperature, I would say that P2 marks the hottest, most rapid, and most constant passage of the prelude and that P1 marks the coldest, least rapid, least dense, and perceptibly calmest section. The musical factors (explained above and summarized below) of the material at and following Golden Section points P1 and P2 contribute to their high contrast, and thus outstanding role, within the prelude (see Figure 6).

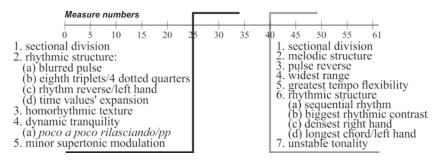


Figure 6: Prelude 2, Golden Section points P1 and P2, diagram and summary

# 6. Prelude No. 3, Op. 8

#### Calculations:

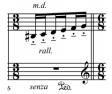
There are **111** bars in this prelude. Three distinct time signatures occur, with 2/4 predominating. Let us calculate the horizontal time line of this prelude in sixteenth notes. Since there are 8 sixteenths in each 2/4 bar, 16 sixteenths in each 4/4 bar, and 24 sixteenths in each 6/4 bar, it follows that the prelude's length **L** is **1104 sixteenths** in total: 736 (92 bars of 8 sixteenths) + 176 (11 bars of 16 sixteenths) + 192 (8 bars of 24 sixteenths). The Golden Section points are **P1**: **682.3** or the **682**<sup>nd</sup> **sixteenth** (1104  $\div \tau \approx 682.324$ ), and **P2**:**421.7** or the **421**<sup>st</sup> **sixteenth** (1104 - 681.324 = 421.676). Within the 111 bars **P1** occurs at the **beginning of bar 78**, on its second sixteenth. **P2** occurs in the **second half of bar 45** on the fifth sixteenth.

# Analysis and Significance Interpretation:

The third prelude is written in through composed *form*, with elements of ternary, rondo, and theme and variations forms, <sup>10</sup> and it consists of: introduction *moderato* (bars 1-4), middle section *allegro* (bars 5-78), transitional *veloce* (79-96), a kind of recapitulation *tempo I* (bars 97-100), and the final *meno mosso* section (101-111). Although widely known for its Ukrainian kolomyika genre features (folk dance and music style), prelude 3 also employs chromatic *harmony*, suspensions, ascending and descending sequences, all expressing romantic aspects of Barvinsky's style. The *texture* of the third prelude is mostly homophonic: with fast continuous sixteenth notes in the melody, accompanied by, mostly, steady eighths and syncopated chords.

**P1** in bar 78 marks a <u>sectional division</u> in that it appears in the final bar of the *allegro* section and immediately precedes the brisk and brilliant *veloce* section. These two sections contrast in character and tempo. The *veloce* section is united by <u>G pedal</u> in the base (this pedal stops in bar 91), and it transfers the music to the *tempo I* section. Note the <u>resemblance</u> with **P2** of prelude 2, where the Golden Section also preceded a section with <u>C‡ pedal</u> and a fast sixteenth-note passage in the right hand, transferring the music into a new section with recapitulation-like material. To see this resemblance, first compare bars 2-6 and 31-35 in prelude 2 with bars 1-4 and 97-100 in prelude 3. Next, compare the transition in bars 30-31 of prelude 2 to the transition in bars 96-97 of prelude 3 (see **Figure 7**).

**a)** Prelude 2, bars 30-31



**b**) Prelude 3, bars 96-97

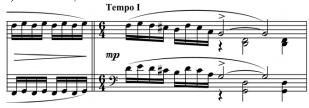


Figure 7: Transitions in prelude 2 and prelude 3

Returning to prelude 3, a <u>tonality shift</u> takes place at the Golden Section point **P1**, from the preceding C minor (see bar 75), to C major (bars 77-78), and to G major (from bar 79). Note that the <u>key signature changes</u> from two flats to single sharp immediately

<sup>10</sup> The elements of ternary form can be seen in the return to introductory material and tempo in bars 97-100, with only slight alterations in harmony and the use of major mode instead of minor. The elements of rondo form are expressed in recurring material of circling or introductory theme throughout the prelude (see bar 5-6). This theme, however, is always more or less altered and transformed, in accordance with the theme and variations form.

after the Golden Section point **P1** (see **Figure 8**). Harmonically, sub-mediant seventh of C major, through seventh of G major, transfers the music to the supertonic harmony of G major. From bar 79, the harmonic structure is ascending in the left hand by six-three chords, specifically the six-three chords of ii, iii, IV, V, flat VI, and flat vii. The ascending chords in the left hand create musical motion and <u>dynamic development</u>. The right hand's passagework adds motor motion and shifting harmonic colours. This section ends with a complete authentic cadence containing a supertonic seventh in its second inversion in bar 89, followed by a dominant seventh in bar 90, and resolved six bars later into the tonic triad. This section is <u>unique</u> in its left hand's <u>rhythmic structure</u>, which consists of 20 subsequent quarters, within bars 79-88, seen nowhere else in the prelude.



Figure 8: Prelude 3, bars 75-82, with grey section at the Golden Section point P1 in bar 78

Looking at the texture, bar 78 is homophonic with chordal syncopated accompaniment and melody. Bar 79 suddenly changes in texture from two to three layers: 1) lowest layer – G pedal or drone in the left hand (each note lasts two bars), 2) middle layer – stepwise ascending triads in their first and second inversions in the left hand, and 3) top layer - fast sixteenth notes descending in sequence, where each sequential unit imitates the 'circling motive' from bars 5-6. The range doubles between bars 78-79 in that bar 78 is 2.5 octaves in range (from G below middle C (small octave) to C of the third octave) whereas bar 79 is 5 octaves in range (from G of the big octave to F# of the fourth octave). Speed doubles as well. Having performed this prelude, and having done research on its previous performances I would say that the contrast in speed is highly outstanding – from ritenuto and natural completion of the phrase in bar 78 to fast and brilliant *veloce* in the following bar. The dynamic reverse happens between these two bars, from gradual decrease to increase in dynamics: bar 78 indicates poco dim., whereas bar 79 starts with *pianissimo* but also indicates *crescendo*. This dynamic increase in bars 79-89 is also achieved by both hands moving from outer extremes of the piano to the middle range. If one were to imagine a curve indicating dynamics, the lowest

(quietest) point of the v-shaped curve would exactly coincide with **P1**. These contrasts in texture, range, and dynamic direction contribute to the division between the two middle sections divided by the Golden Section.

Preceding **P2**, bar 41 concludes the section or episode of bars 32-41. Bars 42-44 recapitulate the initial circling theme of bars 5-7. **P2** occurs in bar 45, marking the sectional division between this recapitulation and the culmination of the following 8 bars, which form the dramatic climax of the prelude (bars 45-53). All the other material in the prelude is calm, dance-like or even exultant; it is often fast but never extremely dramatic. Harmonically, the vicinity of **P2** consists of mostly alternating tonic and subdominant harmonies, with minor dominant and supertonic harmonies. However, immediately following bar 45, harmony also undergoes dramatization with diminished and half diminished chords and chromatic melodic structures. Range doubles from less than 2 octaves (from B flat below middle C to A flat of the second octave) to 4 octaves in bars 45-53 (from C of the big octave to C of the third octave). The music ascends in register around the location of the Golden Section point **P2**. In addition, texture becomes denser in bar 46 (see **Figure 9**).



Figure 9: Prelude 3, bars 42-53, with grey section at Golden Section point P2 in bar 45

The only stable element appears to be the G minor key of this section (from bar 42), except this time <u>G minor tonality</u> does not include E natural, and the colour of the theme is changed – it is pure minor this time (or Aeolian). What this means is that the initial step is half a tone instead of a whole tone (compare the whole tone steps of bars

5, 12, and 23 with bar 42's half tone step). This return to initial material serves only to prepare for upcoming new material which enters exactly at bar 45! Here the left hand takes the rhythmic figure found in bars 5 and 42 (eighth, quarter, and eighth) and transforms it into an ascending sequence with recurring Gs in the middle and bottom voices. This sequence has increasing dynamics from p and a tempo to a slight crescendo, then poco a poco cresc., another crescendo fork, leading to forte in bar 49. This creates a kind of musical tension, even a dramatic effect. In combination with this, the right hand keeps ascending with the circling theme in a stepwise motion (mostly within the second octave) starting on E flat (bar 45), then on F with B flat tonal base (bar 47), then diatonically shifting the same excerpt a tone higher on G (bar 48), and continuing with yet another shift a minor third higher – on B flat (bar 49). Finally, the second half of the circling theme is heard in bar 50, where E flat plays the role of tonal base, which in bar 51 becomes the starting point of the now downwards diatonically shifted theme. This time the theme arrives at A flat tonal base. The following descending sequence is again a varied fragmentation of the original circling theme. As you can see, apart from ascending direction, stretto is employed, where the second (bar 47), third (bar 48), and fourth (bar 49) sequences in the right hand are twice as short as the first sequence (bars 45-46). Three bars of similar material start at bar 42, implying that the sequence may have initiated in bars 42-44. Then it decreases to two bars 45-46, and, following the Golden Section P2, it reduces again to one bar until bar 49, also reminding of fragmentation. After bar 49 the units expand to two bars. G pedal unites this entire section (bars 42-52). The constant syncopation and ascension, in combination with accents above the syncopated quarters of the left hand create a feeling of unease and tension. In addition, rhythmically strong ascending dotted chords in the left hand (bars 50, 52) add dramatic effect. The accents in the left hand (bars 45, 47-49, and 51) do not figure anywhere else in the prelude, although identical syncopated rhythms do appear. This, as it did at the first point P1, makes the left hand exhibit unique rhythmic structure exactly at **P2** (see **Figure 10**).

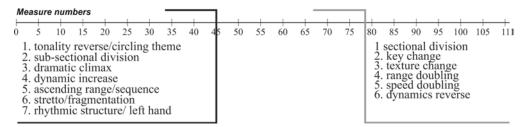


Figure 10: Prelude 3, Golden Section points P1 and P2, diagram and summary

# 7. Prelude No. 4, Op. 8

### Calculations:

Since there are no alterations to the 4/4 time signature, let us view the horizontal time line of this prelude in bars. The length **L** is 84 bars. The Golden Section points are: **P1** at **51.92** or the **end of bar 51** (84  $\div \tau \approx 51.916$ ) and **P2** at the **beginning of bar 32** (84 – 51.92 = 32.08).

# Analysis and Significance Interpretation:

The *form* of this prelude can be interpreted as complex ternary form, with its structural formula being **A** (bars 1-23): **a- a**<sup>1</sup>; **B** (bars 23-44): **b-b**<sup>1</sup>; **A**<sup>1</sup> (bars 45-69): **a**<sup>2</sup>-**a**<sup>3</sup>, and **Coda** (70-84), intertwined with reiterations of excerpts from section **A**. The *tonality* of the prelude contains semitone modulations that often juxtapose whole episodes in half tone key shifts (e.g. B flat to B, D flat to D). The prelude is written in B flat minor. However it ends in B flat major, and modulates on the way to keys such as D flat major, G flat major, F minor, A flat major, D major, and B minor. The *texture* is predominantly homorhythmic, containing independent melodic fragments in middle voices, with homophonic middle **B** section. Much of the *harmony* is standard. At the same time, mediant, minor dominant, flat supertonic, flat submediant, and natural seventh are frequently used. The use of seventh natural scale degree in minor (e.g. bar 1) is a typical harmonic feature of Ukrainian folk choir singing (Arkhimovych et al. 1964, 54). Indeed, the prelude was named *choral* by Barvinsky.

P1 at the end of bar 51 occurs almost on a bar line, immediately preceding the prelude's most intense but exuberant culminating section. Bar 51 in section  $A^1$  ends its direct similarity to section A. The end of bar 51 (final two eighth notes) already initiates this important and functionally outstanding section of the prelude (bars 52-65 or even to 69). The transition in bars 51-52 is similar to that in bars 16-17, but this time the second bar introduces higher range, tonal shift, key change (to two sharps) expanded length, texture, and harmonic structure. The transition also contains semitone voice leading (bars 51-52, bottom stave, see F - Ab to E - G). Instead of concluding as it does in the first section (a1), this section, after five intense bars of triple forte (the loudest dynamics of the entire prelude), again transfers to an expanded version of material seen in bars 14 and 49, finally developing into new material (bars 57-65). Bar 52 also introduced new material. In terms of tonality around P1, G flat major dominates as the main key between bars 48 and 51. Both the presence of Cb and the harmonic progression of these bars confirm the G flat major tonality. From this perspective it becomes clear that the dominant ninth of bar 51 does not resolve into the expected G flat major tonic harmony, but rather it shifts to D major dominant harmony. From yet another, less obvious and reasonable perspective, the shift functions as a sudden semitone key shift. The key signature and reappearing tonic, as well as dominant of D flat major, more readily signify D flat major in these bars. Bars 49-51 could imply A flat minor tonality. The first chord of bar 49 contains A flat minor triad, followed by A flat minor seventh. The next two bars contain F\$ and a harmonic progression that does not confirm the A flat minor as a key, whereas the latter can be considered the minor dominant of D flat major. In this case, the final chord of bar 51 is the minor dominant of D flat major, with added sixth (F in the left hand). Bar 52 starts the dominant seventh of D major, resolving into the tonic on the third beat of this bar. This tonality shift  $(\underline{D})$ flat major to D major) highlights P1's function of transitioning into the prelude's climax (see Figure 11).



Figure 11: Prelude 4, bars 47-54; P1 in bar 51, and semitone tonality shift in bars 51-52

P2 in bar 32 occurs exactly in the middle of the middle section B, marking a subsectional division from  $\mathbf{b}$  to  $\mathbf{b}^1$ . The two subsections of the middle section are highly identical, with six-bar-long main sentences (bars 25-30 for **b** and bars 33-38 for  $b^1$ ), where the sixth bar of each sub-section differs and acts as a transition. Two bars 31-32 form a transition into the second subsection. P2 in the beginning of bar 32 (32.08, or 32.1) is in the middle of this transition and, thus, of section **B**. The two middle subsections are again juxtaposed in another ascending semitone shift of material and tonality. **P2** also coincides with this shift through another key change (to two sharps) marked with semitone voice leading (bars 32-33, D to C#, A to A#, and F to E). Tonality and *harmony* structures are significant at this point. From B flat minor, the music modulates to B minor through the secondary dominant of B minor's dominant. Bar 32 contains F minor tonic chord in its first inversion, followed by an enharmonic pivot chord, where the submediant seventh of F minor equates to B minor's secondary dominant, or F sharp major's dominant seventh. Bar 33, thus, initiates with the dominant seventh chord of B minor. In bar 35 it then transfers to A major, next to B flat major (bar 37), the key signature of which recurs in bar

43. The two subsections of the **B** section also contrast in dynamics and function. Whereas the first subsection **b** is *piano* in *dynamics* with indications *tranquillo* and *la melodia ben cantado*, ending with *poco rit*. (bar 32), the second subsection **b**<sup>2</sup> starts with *a tempo* and *poco a poco cresc*. and concludes with *forte*, gradually transferring to *fortissimo* (*ff*), and concluding with *poco allargando* without dynamic decrease. Indeed, the second subsection of the middle section initiates a transition or preparation for culmination of the prelude, as the next section keeps increasing dynamically to *sempre ff* (bar 45), *cresc*. (bar 51), and *fff* in bar 52. In any case, the second Golden Section point coincides with a <u>reverse</u> of the initially calm middle section to gradual intensity, <u>dynamics</u>, chromaticity, and texture buildup.

To compare and summarize, both Golden Section points are significant to the tonal structure of the prelude in that they occur at transitions to new tonal centers, almost exactly prior to establishment of new keys. Both P1 and P2 signal half tone ascending shift from five flats to two sharps. In bars 32-33 (P2) it is minor tonality that predominates, whereas in bars 51-52 (P1) it is major tonality. The two points contrast in dynamics as well, from piano in bars 32-33 to cresc. and fff triple forte in bars 51-52. As in prelude 1, the Golden Sections in prelude 4 occur at points which contrast in function. The earlier point P2 is tranquil, while the later point P1 is highly intense and very loud. Notable is also the textural contrast between the two points. The earlier point P2 occurs in the middle of homophonic texture with clear melodic line and accompanying figure in the left hand, whereas the later point P1 is homorhythmic, with wide chords of higher range. While the earlier point is solemn in its melodic material, the later point appears as the prelude climaxes with almost heroic melody. It also initiates imitation of Ukrainian church bells (see the two A-octaves in bar 52). 11 The melodic material of the middle section is characteristic of Ukrainian folk tune. Moreover, that exact melodic line echoes the first prelude's initial melodic line, the theme of prelude 1. The vocal quality of this melody (see prelude 1 bars 1-10) and its distinct character have led a number of musicologists to associate it with Ukrainian vocal folklore (Nazar 2000, 18). As in prelude 1, the two Golden Section points in prelude 4 contrast in texture (homophonic vs. homorhythmic), dynamics (very soft vs. triple forte), tonal colour (minor vs. major), and range (D flat of the big octave to D of the second octave in bar 32, vs. C flat of the small octave to C flat of the third octave in the second half of bar 51). The similarities shared by preludes 1 and 4 in melodic material and function of Golden Section points unifies these two among Barvinsky's piano preludes. See Figure 12 for a diagram and summary of Golden Section points in prelude 4.

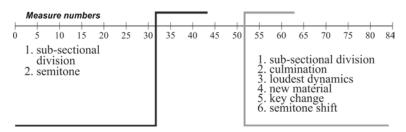


Figure 12: Prelude 4, Golden Section points P1 and P2, diagram and summary

### 8. Conclusion

In conclusion, all four preludes prove to contain musically outstanding material at, immediately preceding, or immediately following Golden Section points. In very general terms, Golden Section points coincide with the first prelude's climax as well as its most tranquil point; with the second prelude's points of largest range, textural, and rhythmic contrast; with the third prelude's most dramatic and contrasting points; and the

<sup>11</sup> This is not unique, as this feature will reappear within the cadential material where B-flat octaves again imitate this traditional sound.

fourth prelude's climax as well as its textural contrast point. Also, key change, semitone shift in tonality, and rhythmic flow interaction often surround Golden Section points in Barvinsky's preludes 1-4.

Many visual artists, architects, and sculptors were known to have consciously employed Golden Section proportions in their work. Despite their prevalence in his work, I have found no documentation suggesting Barvinsky's deliberate use of them, nor is there mention in any literature available to the current author of any connection between Barvinsky's works and this mathematical proportion. Bachmann said of Bela Bartok, "Essentially it seems that Bartok's use of the Golden Mean and Fibonacci numbers simply clarifies and focuses the exceptional balance and proportion that exist in his music" (Bachmann 1979, 80). Could the same be said of Barvinsky, or should one search for clearer explanations?

The true aim of the above study is not to prove Barvinsky's knowledge of mathematics, or laws of nature and physics, but to illustrate that he may be classed among outstanding composers and talented musicians, since the Golden Section also figures in works of great composers like Bach and Chopin (but lacks in works of certain other composers studied by Sabaneev).

I would like to conclude with two postulations. The first postulation is that perhaps the Golden Section, employed unconsciously, may serve as a criterion for quality of art work. Although practically no direct relation between points of folklore imitation and Golden Section were found in Barvinsky's preludes, my second suggestion is that familiarity with folk music knowledge (for which Barvinsky is famous) may be related to a composer's use or perceived importance of the Golden Section, which forms an open question and possibility for further research.

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# Nina Soyfer. The Golden Section in Barvinsky's Piano Preludes Op. 8, Nos. 1-4.

This work is a sui generis harmonic analysis of a few relatively short music compositions. As an example, I took piano preludes of internationally known 20th century Ukrainian composer, Vasyl Barvinsky. The goal of this analysis is to uncover mathematical regularities in the structure of a finished music composition, in particular – how its compositional structure corresponds to Golden Section proportions. Thereby, a confirmation of the above-mentioned correspondence (i.e., mathematical regularity) supports theories of universality of mathematical harmony in the arts.

Keywords: Golden Proportions, Implementation of Mathematics' Harmony in Music

